Lesson 9. Velocity and Acceleration

1 Overview

- How can we find the velocity and acceleration of an object in 3D space?

2 Definitions

- Let $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$ be the position vector an object's position at time t
  - For example, at time $t = 2$, the object is at point
- The **average velocity** of the object over the time interval $[t_1, t_2]$ is
  - Change in position (displacement) per unit time
- The **velocity** of the object at time t is
  - Limit of average velocity as the interval length approaches 0
- The **speed** of the object at time t is
- The **acceleration** of the object at time t is
  - How does the velocity change?

Example 1. Find the velocity, acceleration, and speed of a helicopter at time t with position vector $\vec{r}(t) = \langle e^t, te^t, \ln t \rangle$. What about when $t = 2$?

Example 2. An airplane starts at an initial position $\vec{r}(0) = \langle 0, 1, 0 \rangle$ with velocity $\vec{v}(0) = \langle -1, 1, 0 \rangle$. Its acceleration is $\vec{a}(t) = \langle 6t, 4t, 1 \rangle$. Find its velocity and position at time t.

Hint. If you have $f'(t)$, then what does $\int f'(t)\, dt$ give you?

- In general, we can recover velocity when acceleration is known:

- We can also recover position when velocity is known:

Example 3. An airplane moves in space according to the vector function $\vec{r}(t) = \langle t^2 - 4t, t^2, 4t \rangle$. When is the speed a minimum?